

## Problem 2.25

[Difficulty: 3]

**2.25** Consider the flow field  $\vec{V} = ax\hat{i} + b\hat{j}$ , where  $a = 0.1 \text{ s}^{-2}$  and  $b = 4 \text{ m/s}$ . Coordinates are measured in meters. For the particle that passes through the point  $(x, y) = (3, 1)$  at the instant  $t = 0$ , plot the pathline during the interval from  $t = 0$  to 3 s. Compare this pathline with the streamlines plotted through the same point at the instants  $t = 1, 2$ , and 3 s.

**Given:** Flow field

**Find:** Pathline for particle starting at (3,1); Streamlines through same point at  $t = 1, 2$ , and 3 s

**Solution:**

For particle paths  $\frac{dx}{dt} = u = a \cdot x \cdot t$  and  $\frac{dy}{dt} = v = b$

Separating variables and integrating  $\frac{dx}{x} = a \cdot t \cdot dt$  or  $\ln(x) = \frac{1}{2} \cdot a \cdot t^2 + c_1$

$dy = b \cdot dt$  or  $y = b \cdot t + c_2$

Using initial condition  $(x,y) = (3,1)$  and the given values for  $a$  and  $b$

$c_1 = \ln(3 \cdot \text{m})$  and  $c_2 = 1 \cdot \text{m}$

The pathline is then  $x = 3 \cdot e^{0.05 \cdot t^2}$  and  $y = 4 \cdot t + 1$

For streamlines (at any time  $t$ )  $\frac{v}{u} = \frac{dy}{dx} = \frac{b}{a \cdot x \cdot t}$

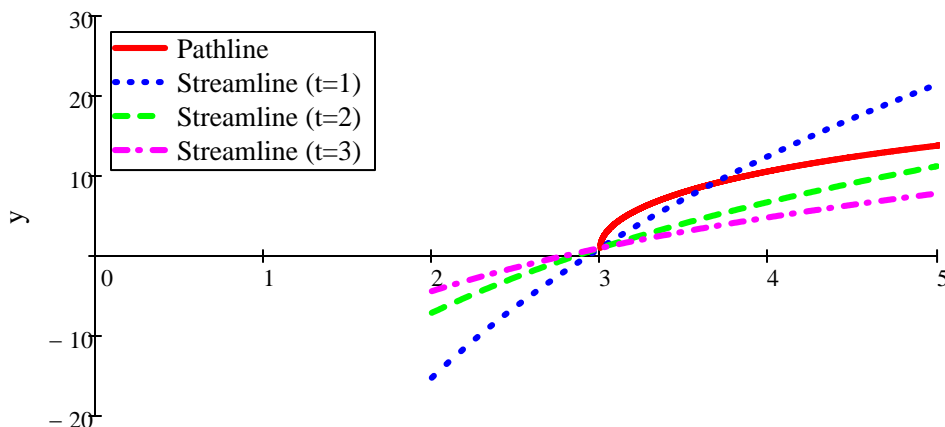
So, separating variables  $dy = \frac{b}{a \cdot t} \cdot \frac{dx}{x}$

Integrating  $y = \frac{b}{a \cdot t} \cdot \ln(x) + c$

We are interested in instantaneous streamlines at various times that always pass through point (3,1). Using  $a$  and  $b$  values:

$c = y - \frac{b}{a \cdot t} \cdot \ln(x) = 1 - \frac{4}{0.1 \cdot t} \cdot \ln(3)$

The streamline equation is  $y = 1 + \frac{40}{t} \cdot \ln\left(\frac{x}{3}\right)$



These curves can be plotted in Excel.